YEDİTEPE UNIVERSITY

MATHEMATICS DEPARTMENT 25TH YEAR SEMINARS

The impossibility of the angle trisection by straightedge and compass revisited

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Abstract: It is well known that the angle trisection or doubling the cube is impossible by using only ruler and compass. The usual algebraic proof uses the fact that if a real number ξ is constructible using only ruler and compass then there is a tower of fields

$$\mathbb{Q} = F_0 \subset F_1 \subset \cdots \subset F_n$$

such that if $n \geq 1$, then $F_i = F_{i-1}(u_i)$ where $u_i \notin F_{i-1}$ but $u_i^2 \in F_{i-1}$ and $\xi \in F_n$. Hence $2^m = [F_n : \mathbb{Q}]$ for some $m \in \mathbb{N}$. This shows that

$$2^m = [F_n : \mathbb{Q}] = [F_n : \mathbb{Q}(\xi)][\mathbb{Q}(\xi) : \mathbb{Q}]$$

So, if the minimal polynomial of ξ in $\mathbb{Q}(x)$ is of odd degree, then ξ is not constructible by only ruler and compass. Moreover, the minimal polynomial of $\cos 20^\circ$ has degree 3. So it is impossible to trisect 60° by using only ruler and compass.

However, this proof requires the fundamentals of vector spaces. In this talk, we will tweak the last part of the proof to avoid vector spaces.

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